Exercise VI, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

1 Let A_1 and A_2 be languages with $A_1 \leq_m A_2$. Is it necessarily true that $\overline{A_1} \leq_m \overline{A_2}$?

2 Classify each of the following languages into one of the following three categories:

decidable, undecidable but recognizable, unrecognizable

Justify your answers with proofs (try reductions).

2a $L_1 = \{\langle M \rangle : M \text{ is a Turing machine that halts on all inputs of length at most } 2025\}$

2b* $L_2 = \{\langle M \rangle : M \text{ is a Turing machine that halts on all inputs of length at least } 2025\}$

2c $L_3 = \{\langle M \rangle : M \text{ is a Turing machine that accepts some string with more zeros than ones}\}$

3 Let A and B be languages. Prove that if all of $A \cap B, \overline{A}$ and $\overline{B}$ are Turing recognisable, then $A \cap B$ is Turing decidable.

4* Consider the language

$$L = \Big\{\langle M \rangle : M \text{ is a Turing machine and } \exists k \in \mathbb{N}\text{: } M \text{ accepts every string of length} \geq k\Big\}.$$

Prove that L is unrecognizable.